Math

Mental discipline

Now, math is just abstract, you may say. There can't be good and evil in math, surely! Two and two make four for everybody... don't they?

Yet even in the teaching of math, there are right and wrong ideas about how we look at children, which is how we look at persons and therefore at the education appropriate to persons.

From the outset, the study of numbers, arithmetic, should provide the opportunity for a study of pattern. A student must learn his tables, but not merely by rote: rather, to prepare the mind for pattern investigations which strongly depend on the awareness of numbers and of number relationships. Learning just by rote does not always lead to the awareness of pattern.

For this reason, there is now an intense fight about the importance of learning the tables. Some argue that things that computers can do ought not to take up our children's time, as if number facts might clutter the mind. Others insist that tables are basic, and then enforce a rote memory that does not function very well outside the little box of school math dominated by sales transactions; this does amount to a kind of clutter because sales transactions are not a significant part of most of our lives.

The various number tables must indeed be mastered, both as number facts and as awareness of quantity and pattern. All the little tricks like casting out nines are worthwhile because of the patterns they present; and Cuisenaire rods and domino games are worthwhile for the same reason. Other manipulatives that make use of the abacus or of geometric constructions so as to invite the investigation of squares, triangular numbers, other arrays or prime numbers, are all valuable. Students enjoy them as puzzles, and this is exactly how the real mathematicians first enjoyed them.

In due time, at least by high school, the study of geometry should be an exercise, not only in the construction of lines, arcs, and angles, but in logic, in the construction of a small system of assumptions and then of numerous theorems that build on them. Such a network of ideas, built upon a minimum set of assumptions, teaches one how completely everything changes when just one basic assumption changes. This exercise, which is especially associated with the study of Euclid, is appropriate for Catholics who recognize their responsibility to develop the capacity of the mind to generate a clear organization of religious thought around a minimum collection of doctrinal ideas. The student observes that rejecting a single portion of a tight network of ideas may well imply the unraveling of the entire network. This is the fate of theology infected by heresy.

However, it must be made clear that Euclid is math, not philosophy. Self-consistency is not the same thing as truth. The pagans, the Gnostics, seeing that they cannot believe in Truth, employ Euclid for a philosopher; seeking only consistency; this is not a sufficient wisdom.

Pattern for physics

Finally, various advanced disciplines of math repeatedly provide startlingly useful metaphors for physics, assisting us in our conquest of the physical world. In view of the mandate in Genesis 1 – "Increase and multiply and fill the earth and subdue it" – the culture of science and of technological invention is an important response to God's will, and it absolutely depends upon the conquest of mathematics.

All of these mental experiences are thus valuable in the formation of a disciplined mind ready to engage the physical universe. In the curriculum as a whole, the influence of the Incarnational perspective will constantly make itself felt in the willingness of Christians to engage the physical world with confidence and effectiveness.

More pattern notes

Because Islamic law forbids the making of images (like the First Commandment of Moses, only interpreted more literally than we do) the decor of a mosque is often composed of tile patterns rather than faces or scenes. Such patterns can be very complicated, and very beautiful, often with

an advanced use of tessellations. A tessellation is a repeating geometric figure that completely covers a plane. A checkerboard is a tessellation of squares. It is also easy to make a tessellation of triangles. It is impossible to tessellate regular pentagons.

A more complicated type of tiling was envisioned by the mathematician Roger Penrose, who found sets of shapes which can completely cover a plane surface, but without actually repeating any pattern. One Penrose pair is simply a triangle and a diamond with specific angles.

All such patterns help to form the mathematical imagination.

A Bag of Tricks or a drudgery

Too often math is presented either as a bag of tricks for the clever to memorize, or as a drudgery happily to be replaced by machines. As a bag of tricks, math enables super-competent students to do fun things but not to take them seriously. Meantime the average student is all too happy to leave the tedious grind of numbers to a machine. The whole enterprise of mathematical competence is trivialized when it stands between the computer and the wizard's hat.

This is a crucial issue. Math education is the education of God's children for the following of God's will. Many modern textbooks dismiss all memory work as an outmoded waste of time. As a result, students are not prepared for the study of either pattern or logic, nor will their generation be able to advance the culture of technological invention, which requires a confident, informed, and disciplined mind.

Notice, however, that the culture of death has no advantages over the Incarnation. The "bag of tricks" is available to the educated man and it delights him. But we must "seek first the kingdom of heaven," doing those things which are right in themselves. Then everything else will be added.

Philosophy & Gödel's theorem

One particularly exciting philosophical development in mathematics is Gödel's theorem, a mathematical statement of the inescapable incompleteness of any conceivable system of postulates, even in math or logic. What Gödel discovered and proved was that no matter how you construct such a system, there will always be a portion that cannot be proved, that is given or assumed; a necessary starting place, not subject to proof. From this it follows that the physical universe, even understood as a tight mathematical system, cannot be closed to surprises, for the assumptions may always contain unexpected dimensions. Such freedom from necessity in the realm of physics, in turn, opens up even to logic the essentially poetic nature of the physical universe. It is not an "a priori" universe – a universe that had to be this way; when our faith teaches that the universe is as it is simply because God, in His freedom, willed it so, that is perfectly reasonable.

I realize that this idea must be somewhat murky, given such brief expression, but I include it because it underlines a philosophical point that precisely opposes a purely mechanistic concept of the universe. From time to time, someone will make the claim that if the universe obeys mathematical laws, it cannot be open to creativity or free will; these must be mere appearances in a purely mechanical and determined system. Not so, says Gödel. No matter how much the physics of the universe is based on math, it still has room for something as yet undefined, even for creativity and freedom. If you have a student who loves math and begins to slide into determinism, encourage him to study Gödel in support of his faith.